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Spectral characterizations of graphs

Spectral graph theory deals with the relation between the structure of a graph and the eigenvalues (spectrum) of an associated matrix, such as the adjacency matrix A and the Laplacian matrix L. Many results in spectral graph theory give necessary condition for certain graph properties in terms of the spectrum of A or L. Typical examples are spectral bounds for characteristic numbers of a graph, such as the independence number, the chromatic number, and the isoperimetric number. Another type of relations are characterization. These are conditions in terms of the spectrum of A or L, which are necessary and sufficient for certain graph properties. Two famous examples are: (i) a graph is bipartite if and only if the spectrum of A is invariant under multiplication by -1, and (ii) the number of connected components of a graph is equal to the multiplicity of the eigenvalue 0 of L. In this talk we will survey graph properties that admit such a spectral characterization. In the special case that the graph itself is characterized by the spectrum of A or L, we say that the graph is determined by the considered spectrum. It is conjectured that almost all graphs are determined by their adjacency spectrum (and perhaps also by the Laplacian spectrum). We will report on recent results concerning this conjecture.