SALEM GRAPHS

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The spectrum of a graph is the spectrum of its adjacency matrix. Salem graphs are graphs whose all eigenvalues but at most two lie in the segment $[-2, +2]$, namely one for non-bipartite graphs and two (one positive the other negative) for bipartite graphs. The interest for studying Salem graphs stems from the fact that each Salem graph gives rise to a Salem number.

As well known in the spectral graph theory, graphs with least eigenvalue not less than -2 are generalized line graphs, or exceptional graphs arising from the root system E_8. If in addition such graphs are reflexive (have the second largest eigenvalue not greater than $+2$) then they are Salem graphs, and also non-bipartite ones. Bipartite Salem graphs are just reflexive bipartite graphs (recall, their spectrum is symmetric with respect to the origin).

In our investigations we put focus on reflexive line graphs. Then we first consider the spanning trees of their root graphs, and by considering the reflexivity of their subdivision graphs (as an easier task) we were in position to decide if the line graph of some tree is a Salem graph. By adding edges to such trees we next consider multi-cyclic graphs whose line graphs are reflexive. Besides theoretical arguments, a computer search is used in our investigations. As a curiosity, some of the Salem graphs being identified are not only too big, but also unbounded in order.

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