Jorge Orestes Cerdeira,
Dep. Matemática and Centro de Matemática e Aplicações, Faculdade de Ciências e Tecnologia, Universidade NOVA de Lisboa

On the scheduling of periodic events

How should N regular polygons be inscribed on a circle so that the minimum distance between two adjacent vertices is as large as possible? How should trains that arrive at a railway station in constant intervals a_i be scheduled so that the safety interval between two trains is maximum? These are different specifications of the same problem: Given a collection A of N (possible repeated) positive integers a_i, find $\delta_i \geq 0$, for $i = 1, \ldots, N$, such that $z = \min |m\, a_i + \delta_i - (n\, a_j + \delta_j)|$, for $i, j = 1, \cdots, N, i \neq j$ and $m, n \in \mathbf{N}_0$ is maximum.

We discuss the problem, and consider the restricted version where for every pair $i, j = 1, \ldots, N, i \neq j$, $|\delta_j - \delta_i| \leq \gcd(a_i, a_j)$, the greatest common divisor of a_i and a_j. We present Mixed Integer Linear Programming (MILP) formulations, describe a procedure to obtain bounds on maximum z, and report computational results. For the restricted version we give a combinatorial procedure that finds optimal solutions, compare the performance of this approach with the MILP model, and prove that the problem is NP-hard.

Joint work with Ricardo Saldanha, SISCOG and Pedro Cristiano Silva, Univ of Lisbon.